in particular, for $b_1 = b_2 = 0$ ($\alpha_1 = a_1$, $\alpha_2 = a_2$, $\nu = 0$) it follows from (35) that

$$q_i(t) = q_{io}\left[1 - \frac{2}{\pi}\operatorname{arctg}\sqrt{\frac{t}{t_1}}\right] \quad (i = 1,\ 2). \tag{36}$$

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TEMPERATURE MEASUREMENT USING THERMISTOR WITH PULSED OPERATION OF CIRCUIT CONTAINING THERMISTOR AND LINEAR RESISTOR

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A method is considered for determining the basic parameters characterizing a pulsed thermistor–linear resistor temperature-measuring circuit and ensuring increased sensitivity to temperature changes while conserving a given accuracy of measurement.

The main demands imposed on the design of temperature-measuring apparatuses reduce to sensitivity and accuracy. In the event that a semiconductor thermistor is used as the temperature sensor, it turns out that these demands are contradictory, since a high sensitivity of the apparatus requires a significant current flow in the sensor circuit; this current heats up the thermistor and so gives rise to a systematic measurement error. This error is usually reduced at the expense of the sensitivity, by reducing the current, which for microthermistors varies from one to a few tens of microamps.

The dilemma can be obviated to a large degree by pulse operation of the thermistor-containing measuring circuit. If the supply of the $R_T - R$ circuit (i.e., the thermistor–linear resistor circuit) is pulsed in such a manner that the mean power supplied equals the power supplied at dc, then the amplitude of the pulses of supply current or voltage may be increased over the dc value by a factor of $1/\sqrt{\gamma}$ (where γ is the duty factor, the ratio of pulse duration to the pulse repetition period). The heating of the thermistor that occurs in this case too by the current passing through it can be estimated from the curve of the transient process.

The theory of pulse systems is well developed and is presented in detail in Tsypkin's books [1, 2], for example.

Transient processes in thermistor circuits for pulse-type variations of the input quantities are considered in [3-6].

Nonetheless, the practical realization of the pulse method of temperature measurement using semiconductor thermistors has been frustrated until recently due to the absence of a simple and reliable high-speed

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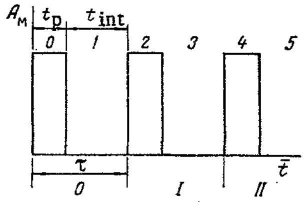

Fig. 1

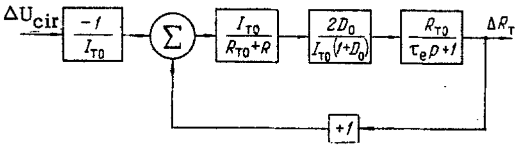

Fig. 2

Fig. 1. Pulses of R_T-R circuit supply voltage. Time t in seconds.

Fig. 2. Structural diagram of R_T-R circuit (input quantity – deviation of circuit supply voltage).

measuring apparatus. This probably explains the absence in the literature of specific circuits and devices with thermistors based on the pulse method, and also the limited number of papers devoted to the underlying theory.

Only in recent times, due to the rapid development of digital measuring techniques, has it become possible to utilize fully the advantages of pulsing the supply of a thermistor–linear resistor measuring circuit. This, in turn, calls for a more detailed study of the transient processes in an R_T-R circuit accompanying a pulse of voltage at its input, with a view to developing methods of determining the basic parameters characterizing the mode of operation of the circuit that would assure high sensitivity and a prescribed accuracy of measurement of temperature.

The present paper is an attempt to solve in part the aforementioned problems.

A stepwise discrete stimulus at the input of a pulse system causes a change in the output quantity. In order to determine the output quantity for a train of square pulses (Fig. 1) acting at the input, we utilize the following equation derived in [1] for an arbitrary pulse repetition period n:

$$Z[n,\ \varepsilon] = X_0 \left\{ K^*(0,\ \varepsilon) - \sum_{\nu=1}^{l} C_{\nu 0} \frac{1 - e^{-q_\nu \gamma}}{1 - e^{q_\nu}} e^{q_\nu (n+1+\varepsilon)} \right\}, \tag{1}$$

where $K^*(0, \varepsilon)$ is the image, in the sense of the discrete Laplace transformation, of the transfer function of an open-loop pulse amplitude modulation (PAM) system, when the transformation parameter

$$q = p\tau \tag{2}$$

equals zero.

As shown in [1], the expressions for $K^*(0, \varepsilon)$ have the following form:

during the time of a pulse, when $0 \le \varepsilon \le \gamma$,

$$K^*(0,\ \varepsilon) = C_{00} + \sum_{\nu=1}^{l} C_{\nu 0} \frac{1 - e^{q_\nu (1-\gamma)}}{1 - e^{q_\nu}} e^{q_\nu \varepsilon}; \tag{3}$$

in the interval between pulses, when $\gamma \le \varepsilon \le 1$,

$$K^*(0,\ \varepsilon) = \sum_{\nu=1}^{l} C_{\nu 0} \frac{e^{q_\nu \gamma} - 1}{1 - e^{q_\nu}} e^{q_\nu (\varepsilon - \gamma)}. \tag{4}$$

The coefficients C_{00} and $C_{\nu 0}$ appearing in expressions (1), (3), and (4) are determined from the equation of the transfer function of the continuous system:

$$C_{00} = \frac{k_p P_C(0)}{Q_C(0)}; \tag{5}$$

$$C_{\nu 0} = \frac{k_p P_C(q_\nu)}{Q'_C(q_\nu)\, q_\nu}. \tag{6}$$

In order to find the transfer function in the present case, when the continuous system comprises the R_T-R circuit, we utilize the structural circuit diagram obtained in [7] on the basis of linearized differential

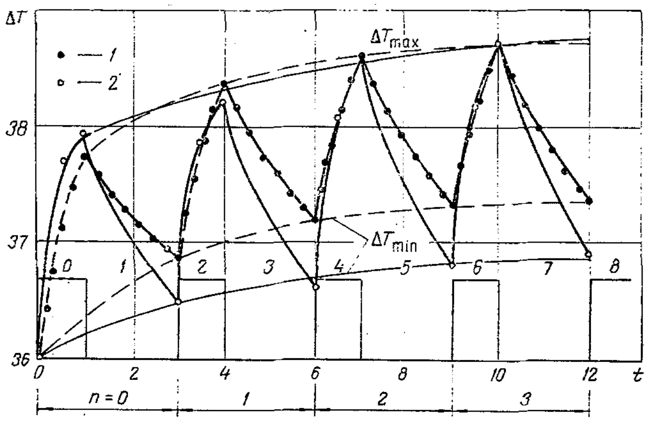

Fig. 3. Transient process in R_T–R circuit for action on input of pulses of supply voltage. 1) Experimental points; 2) calculation. Time t in seconds.

equations describing the electrothermal processes in the circuit. We separate from this structural diagram that part connected with the action at the circuit input of supply voltage variations, after which it takes the form shown in Fig. 2. The transfer function of the continuous system can then be written in the following manner:

$$\frac{\Delta T(p)}{\Delta U_{cir}(p)}=\frac{P_C(p)}{Q_C(p)}=\frac{2U_{T0}}{k(1-D_0\delta)(R_{T0}+R)(\tau_\theta p+1)}. \tag{7}$$

Denoting

$$\alpha=\tau/\tau_\theta, \tag{8}$$

we have that

$$\frac{P_C(p)}{Q_C(p)}=\frac{2U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}\;\frac{\alpha}{\tau p+\alpha}.$$

We introduce the transformation parameter (2), in terms of which the transfer function is finally expressed as

$$\frac{P_C(q_\nu)}{Q_C(q_\nu)}=\frac{2U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}\;\frac{\alpha}{q_\nu+\alpha}. \tag{9}$$

Its characteristic equation has one root. Indeed,

$$k(1-D_0\delta)(R_{T0}+R)(q_\nu+\alpha)=0,$$

whence

$$q_\nu=q_1=-\alpha. \tag{10}$$

Insertion of the obtained values gives for the coefficients C_{00} and $C_{\nu 0}$

$$C_{00}(U)=\frac{k_p 2U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}; \tag{11}$$

$$C_{\nu 0}(U)=\frac{k_p\cdot 2U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}. \tag{12}$$

Since the transfer function of the continuous system has only one pole and the input quantity is voltage pulses of square shape, it follows that $\nu=1$, $k_p=1$ and $X_0=\Delta U_{cir}$.

On the basis of the above, the change in temperature of the thermistor due to the application of a train of pulses at the input of the circuit will be described by the following equations:

during pulses $(0\leq\varepsilon\leq\gamma)$,

$$\Delta T[n,\ \varepsilon]=\frac{2\Delta U_{\text{cir}}U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}\left\{1-\frac{e^{-\alpha\varepsilon}}{1-e^{-\alpha}}\left[(1-e^{-\alpha(1-\gamma)})-(1-e^{\alpha\gamma})\,e^{-\alpha(n+1)}\right]\right\}; \tag{13}$$

in the intervals between pulses ($\gamma \leq \varepsilon \leq 1$),

$$\Delta T[n,\ \varepsilon]=\frac{2\Delta U_{\text{cir}}U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}\,\frac{1-e^{\alpha\gamma}}{1-e^{-\alpha}}\,[e^{-\alpha(n+1)}-1]\,e^{-\alpha\varepsilon}. \tag{14}$$

It can be seen from these expressions that the temperature of the thermistor during a pulse and in the interval between pulses varies in accordance with an exponential law. In the first case the temperature increases; in the second, it decreases. The overall character of the variation of the output quantity of the pulse system can be expressed with the aid of the so-called "envelope" [1], the curve joining the maximum or minimum values of the output quantity in the transient process. The maximum values of $\Delta T[n, \varepsilon]$ over the given repetition periods are attained at $\varepsilon=\gamma$ and the minimum values, at $\varepsilon=0$. Inserting these values of ε into (13) gives the equations of the envelopes:

$$\Delta T_{\max}=\Delta T[n,\ \gamma]=\frac{2\Delta U_{\text{cir}}U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}\left\{\frac{(1-e^{-\alpha\gamma})\,[1-e^{-\alpha(n+1)}]}{1-e^{-\alpha}}\right\}; \tag{15}$$

$$\Delta T_{\min}=\Delta T[n,\ 0]=\frac{2\Delta U_{\text{cir}}U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}\left\{\frac{(e^{\alpha\gamma}-1)\,(1-e^{-\alpha n})\,e^{-\alpha}}{1-e^{-\alpha}}\right\}. \tag{16}$$

With increasing n, the envelopes also vary in accordance with an exponential law.

As $n\rightarrow\infty$, a state of dynamic equilibrium sets in in the pulse system, and the envelopes tend to constant values corresponding to the maximum and minimum values of the steady-state (quasistationary) temperature:

$$\Delta T_{\max\,ss}=\Delta T[\infty,\ \gamma]=\frac{2\Delta U_{\text{cir}}U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}\,\frac{1-e^{-\alpha\gamma}}{1-e^{-\alpha}}; \tag{17}$$

$$\Delta T_{\min\,ss}=\Delta T[\infty,\ 0]=\frac{2\Delta U_{\text{cir}}U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}\,\frac{(e^{\alpha\gamma}-1)\,e^{-\alpha}}{(1-e^{-\alpha})}. \tag{18}$$

It is normally assumed that the steady-state value of the output quantity is achieved in practice after that number of repetition periods n for which the envelopes differ from the respective steady-state values by not more than 5%. This condition can be used to determine the end of the transient process. In our case

$$\frac{\Delta T_{\min\,ss}-\Delta T_{\min}}{\Delta T_{\min\,ss}}=e^{-\alpha n}\leqslant 0.05,$$

which corresponds to

$$n\geqslant\frac{\ln 0.05}{\alpha}\approx\frac{3}{\alpha}, \tag{19}$$

from which it follows that the greater $\alpha=\tau/\tau_\theta$, the shorter the duration of the transient process. If $\alpha\geq 3$ ($\tau\geq 3\tau_\theta$), the transient process terminates in the first cycle.

Figure 3 shows the character of the temperature variation of the MT-54 thermistor and envelopes plotted from experimental data. For comparison we also show in this figure the calculated curves obtained from the above formulas.

In order to find the remaining parameters characterizing the pulsed mode of operation of an R_T–R circuit we proceed from an analysis of Eqs. (13) and (14).

Suppose a unit step of voltage acts on the input of the system. For this condition let us determine the time during which, for a given step amplitude ΔU_{cir}, the thermistor will not be overheated by more than a previously prescribed permissible value $\Delta T_{\text{heat per}}$, determined by the specified accuracy of measurement.

For n = 0 and $\gamma=1$, which correspond to this condition, Eq. (13) acquires the form

$$\Delta T[0,\ \varepsilon]=\frac{2\Delta U_{\text{cir}}U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}\,(1-e^{-\alpha\varepsilon}). \tag{20}$$

Setting $\Delta T[0, \varepsilon] = \Delta T_{\text{heat per}}$, we obtain from (20) after some manipulation

$$-\alpha\varepsilon = \ln\left[1 - \frac{\Delta T_{\text{heat per}}\, k(1-D_0\delta)(R_{T0}+R)}{2\Delta U_{\text{cir}} U_{T0}}\right],$$

from which, remembering the adopted notation, we have

$$t_p = -\tau_\theta \ln\left[1 - \frac{\Delta T_{\text{heat per}}\, k(1-D_0\delta)(R_{T0}+R)}{2\Delta U_{\text{cir}} U_{T0}}\right]. \tag{21}$$

In this manner, we see that the time (starting from the beginning of the step) during which the overheating of the thermistor does not exceed a previously prescribed value depends on the amplitude of the voltage step, the thermal time constant of the circuit τ_θ, and also on the position of the initial working point on the static volt–ampere characteristic of the thermistor. For a pulse amplitude equal to the amplitude of the step, this time can be taken as the pulse duration t_p.

The duration of the interval between pulses will also have a significant effect on the accuracy of measurement, since it determines the time at which the thermistor changes over to the initial mode of operation at the end of each repetition period.

In the theory of pulse systems one normally assumes that if t_{int} is greater than four times the time constant of the system, then in effect the system is subjected to a single pulse. However, for a chosen input power to the R_T-R circuit and pulse duration, excessive increase of the interval between pulses will lead to an irretrievable loss of information on the temperature variation of the medium under investigation unless its choice is dictated by other considerations. At the same time, excessive reduction of t_{int} will cause the initial working point on the volt–ampere characteristic to vary, with consequent accumulation of systematic measurement error.

Prescribing the residual permissible overheating of the thermistor at the end of the repetition period, let us determine the corresponding t_{int} starting from a known pulse duration and input power.

For $n=0$ and $\varepsilon=1$, Eq. (14) acquires the form

$$\Delta T_{\text{cool per}} = \frac{2\Delta U_{\text{cir}} U_{T0}}{k(1-D_0\delta)(R_{T0}+R)}(e^{\alpha\gamma}-1)\,e^{-\alpha},$$

from which we obtain the following after some manipulation and replacing α by its value from (8):

$$\tau = -\tau_\theta \ln\left[\frac{\Delta T_{\text{cool per}}\, k(1-D_0\delta)(R_{T0}+R)}{2\Delta U_{\text{cir}} U_{T0}(e^{\alpha\gamma}-1)}\right]. \tag{22}$$

Further, inserting into (22) $\alpha\gamma = t_p/\tau_\theta$ and the value of t_p from (21), we have, finally,

$$t_{int} = \tau - t_p = \tau_\theta \ln \frac{\Delta T_{\text{heat per}}}{\Delta T_{\text{cool per}}}. \tag{23}$$

Equations (21) and (23) are important in practice as they give the pulse duration t_p and the interval between pulses t_{int} as functions of the power supplied to the thermistor, the character of which is shown in Figs. 4 and 5 for the MT-54 thermistor.

The amount by which the power supplied to the thermistor in pulsed operation of the R_T-R circuit exceeds the power which could be supplied in continuous operation is determined by the coefficient of thermal loading. Its value is found from (17):

$$\eta = \frac{2\Delta U_{\text{cir}} U_{T0}}{\Delta T_{\text{max ss}}\, k(1-D_0\delta)(R_{T0}+R)} = \frac{1-e^{-\alpha}}{1-e^{-\alpha\gamma}}. \tag{24}$$

For $\gamma=1$, which corresponds to continuous operation of the circuit, $\eta=1$. With decreasing γ the coefficient of thermal loading increases.

In conclusion, on the basis of the above analysis we can suggest the following order in which to select the basic parameters characterizing the pulsed mode of temperature measurement using an R_T-R circuit:

1) Starting from the prescribed accuracy of measurement, establish with a certain safety margin the permissible overheating of the thermistor in the time of action of the first pulse $\Delta T_{\text{heat per}}$.

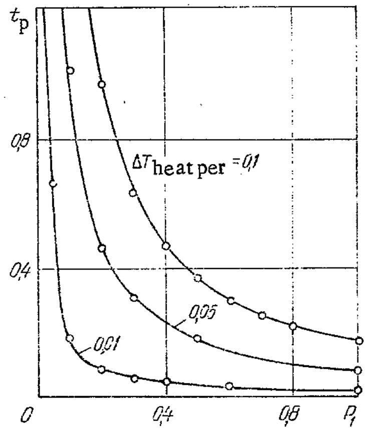

Fig. 4

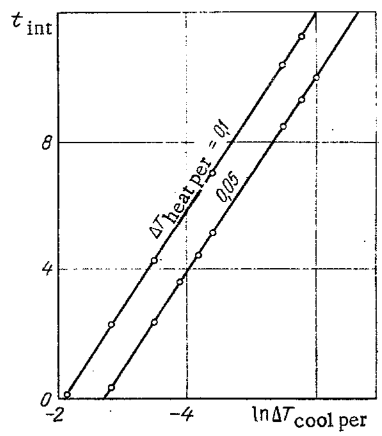

Fig. 5

Fig. 4. Pulse duration t_p as function of input power for various values of thermistor overheating. Time t (sec); power P (mW).

Fig. 5. Interval between pulses t_{int} as function of permissible residual overheating for various values of overheating within pulse.

2) From the prescribed sensitivity of the measuring device, determine the power (voltage) supplied to the circuit in the pulse, and calculate via (21) the pulse duration assuring at this power the established overheating.

3) Prescribing the residual permissible overheating of the thermistor at the end of the interval between pulses $\Delta T_{cool\ per}$, determine via (23) t_{int}.

4) Having determined the repetition period of the pulses $\tau = t_p + t_{int}$, find from (8) the quantity α and from (19) n and the time at which the transient process ends.

5) Inserting the obtained values into (17) and (18), determine the maximum value of the thermistor overheating in the time of the pulse $\Delta T_{max\ ss}$ and in the interval between pulses $\Delta T_{min\ ss}$ at the end of the transient process.

If the obtained $\Delta T_{max\ ss}$ and $\Delta T_{min\ ss}$ exceed the overheating of the thermistor permissible from considerations of accuracy, the calculation must be repeated after a smaller initial overheating has been established or by varying the values of the power in the pulse, t_p, and t_{int}.

Considerations of the most efficient use of the possibilities of an existing measuring apparatus may immediately prescribe the optimum value of the pulse duration and in multipoint temperature measurement, the interval between pulses as well. In this case it remains to determine using the above relationships only the power supplied to the circuit in the pulse which assures the permissible thermal overload of the thermistor.

NOTATION

R_T, thermistor resistance; R, resistance of linear resistor; $\gamma = t_p/\tau$, duty ratio; t_p, pulse duration; $\tau = t_p + t_{int}$, pulse repetition period; t_{int}, interval between pulses; n, arbitrary pulse repetition period; Z, output quantity of PAM system; $\varepsilon = \Delta t/\tau$, relative time, varying from 0 to 1; Δt, time measured from the beginning of each pulse; X_0, amplitude of input excitation; p, Laplace operator; $k_p = A_M/X_0$, amplification coefficient characterizing pulse element of system; $P_C(q_\nu)$, $Q_C(q_\nu)$, numerator and denominator of transfer function of continuous system; $Q'_C(q_\nu)$, derivative of denominator of continuous system; q_ν, roots of equation $Q_C(q_\nu) = 0$; $P_C(0)$, $Q_C(0)$, numerator and denominator of transfer function of continuous system for $q_\nu = 0$; ΔT, change of thermistor temperature due to change of circuit supply voltage ΔU_{cir}; U_{T_0} initial voltage across thermistor at working point on static volt–ampere characteristic; R_{T_0}, initial resistance of thermistor at working point; $\delta = (R_{T_0} - R)/(R_{T_0} + R)$, dimensionless parameter; k, static dissipation factor of thermistor; D_0, dynamic factor [7]; τ_0, thermistor time constant; $\tau_\theta = \tau_0/(1 - D_0\delta)$, time constant of $R_T - R$ circuit; $T_{max\ ss}$, $T_{min\ ss}$, respectively, the maximum and minimum quasistationary thermistor temperatures established in the transient process; $\Delta T_{heat\ per}$, the permissible temperature to which the thermistor may be overheated in the time of a pulse; $\Delta T_{cool\ per}$, the permissible residual overheating of the thermistor at the end of the interval between pulses; η, thermistor coefficient of thermal loading; τ_e, electrical time constant of thermistor; ΔR_T, increment of thermistor resistance due to action of voltage pulse.

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DETERMINATION OF THE MEAN ENERGY DENSITY OF A LIGHT BEAM IN AN IRREGULAR THERMODYNAMIC LIGHT GUIDE

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UDC 654.91:621.372.81.09:535.811

The mean energy density of a beam of light in an irregular thermodynamic light guide with random lens shifts is calculated using the approach described in [3].

§ 1. The energy structure of a beam of light propagating in an optical communication line consisting of a series of discrete phase correctors is usually studied by quasioptical methods [1]. For regular lines this approach enables one to obtain fairly complete information on the mode structure, the losses in the line, etc., but serious difficulties are encountered when one attempts to apply a similar analysis to lines which have a different kind of statistical irregularity (displacement of correctors, rotation of the beam, differences between the corrector parameters, etc.).

Geometrical optics, which is simpler than other approaches, enables one to obtain reasonable information on the energy distribution in the beam of light. One of the versions of this approach is the ray method (see [2] and the references given there). Another approach by which the energy distribution can be analyzed using geometrical optics has been described in [3]. A differential equation for the light energy density was obtained there which enables one to find the energy density at any point in the region for an assigned initial distribution. The evolution of the energy distribution of the beam in this approach is traced in phase space of the beam, and the energy density is therefore a function of the vector which defines the coordinates of the point in space and the vector which defines the direction at the same point. The main result obtained in [3] is that the energy density $U(x, p_i, q_i)$ at a point M with coordinates (x, q_i) in the direction (p_i) is determined by the initial energy

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